Balanced Trees

- in an ideal world we would like to always ensure that our binary trees are perfectly balanced
- keeping a tree perfectly balanced after each deletion and insertion is computationally expensive

AVL Trees

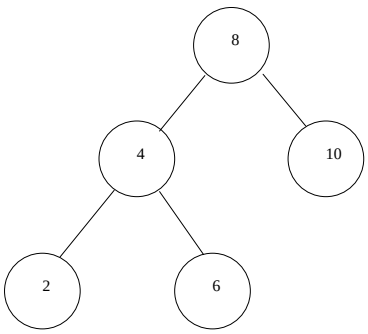
- Adelson-Velskii and Landis postulated a more relaxed definition for balancing a tree
  - a tree is balanced if and only if for every node the heights of its two subtrees differ by at most 1
- this provides a compromise between efficient tree access and the need to rebalance

AVL Trees

- consider insertion of a new `int` on the `left` branch, causing the `left` height to increase by one
- here we have three cases
  - `height(left) == height(right): left` and `right` become of unequal height, but the balance criterion is not violated
  - `height(left) < height(right): left` and `right` become of equal height, balance criterion was improved
  - `height(left) > height(right): left` and `right` differ by 2, balance criterion was violated, the tree must be balanced

AVL Trees

- consider adding `1`, `3`, `5`, `7`, `9` and `11`



- which numbers require the tree to be rebalanced?

AVL Trees

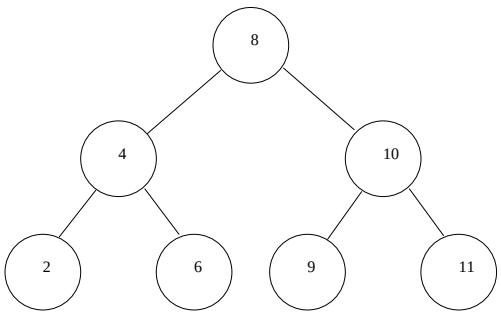
- in summary we need to rebalance the tree if the left and right height differ by two or more

Rebalancing a tree

- rebalancing a tree is achieved through two operations which might be applied a number of times
  - rotate right
  - rotate left

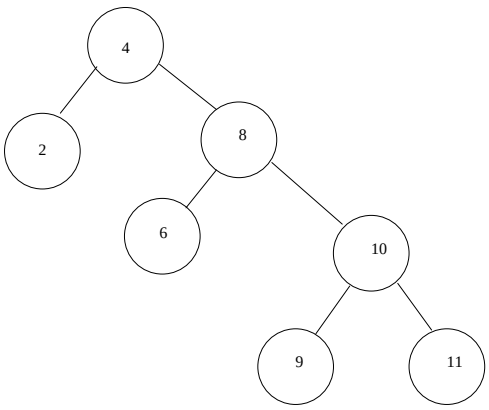
Rotate right

-



Rotate right

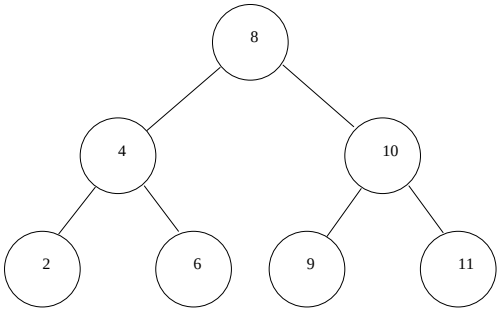
-



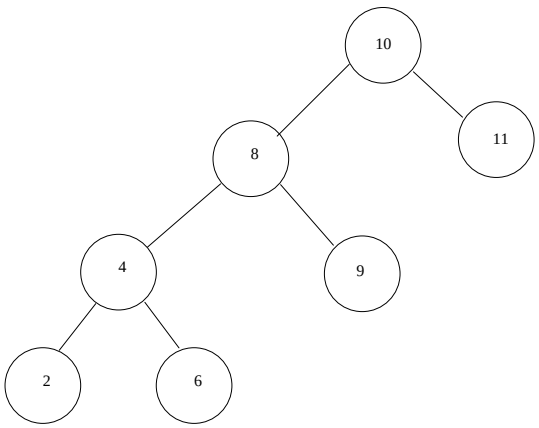
Rotate right axioms

```
rotate_right (empty ())      -> empty ()
rotate_right (cons (i, l, r) -> cons (root (l),
                                      left (l),
                                      cons (i, right (l),
```


Rotate left



Rotate left



Rotate left axioms

```
rotate_left (empty ())      -> empty ()
rotate_left (cons (i, l, r) -> cons (root (r),
                                     cons (i, l, left (r)
                                     right (r))
```

rotate_right

examples/c++/trees/avl/int/tree.cc

```
/*
 *  rotate_right - pre-condition : left branch is not empty.
 *                 post-condition: a new tree is return which
 *                                 is the result of rotating the
 *                                 current tree.
 */

tree tree::rotate_right (void)
{
  if (is_empty ())
    return *this;
  else
    return cons (left ().root (),
                 left ().left (),
                 cons (root (),
                       left ().right (),
                       right ()));
}
```


rotate_left

examples/c++/trees/avl/int/tree.cc

```
/*
 *  rotate_left - pre-condition : right branch is not empty.
 *                post-condition: a new tree is return which
 *                                is the result of rotating the
 *                                current tree.
 */

tree tree::rotate_left (void)
{
  if (is_empty ())
    return *this;
  else
    return cons (right ().root (),
                 cons (root (),
                       left (),
                       right ().left ()),
                 right ().right ());
}
```


insert

examples/c++/trees/avl/int/tree.cc

```
tree tree::insert (int i)
{
  if (is_empty ())
    return cons (i, empty (), empty ());
  else
    return insert_non_empty (i).balance ();
}
```


balance

examples/c++/trees/avl/int/tree.cc

```
tree tree::balance (void)
{
  tree r = right ();
  tree l = left ();
  int  d = l.height () - r.height ();
  tree t = *this;

  if ((d == 0) || (d == 1) || (d == -1))
    return t;
```

balance

- **examples/c++/trees/avl/int/tree.cc**

```
  /*
   *  now rotate
   */

  if (d < 0)
    return t.rotate_left ();
  else if (d > 0)
    return t.rotate_right ();

  return cons (t.root (),
               r.balance (),
               l.balance ());
}
```


Conclusion

- a functional approach can be useful in describing rotate right and left
- a functional approach allows a balance function to be implemented reasonably simply
- at the expense of lost memory, hence the requirement for a garbage collector