Binary trees

- a binary tree is a finite set of nodes which is either empty or consists of a data item (called the `root`) and two disjoint trees called `left` and `right` subtrees
- recall with the list implementation, there was no easy and fast method for in order insertion
    - conversely binary trees allow for very fast in order insertion
- peculiarly the data structure declaration for a binary tree resembles that of a double linked list
    - the access mechanisms might also be very similar

Properties of binary trees of integers

- are traditionally organised such that:
  - left branch contains values < root value
  - right branch contains values >= root value

- the root value here is the value at a possible node (or subtree)
  - not the value at the top of the complete tree

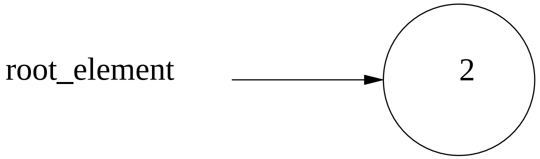
Example 1

- let us add: `2`, `3`, `1` and `4` to a binary tree

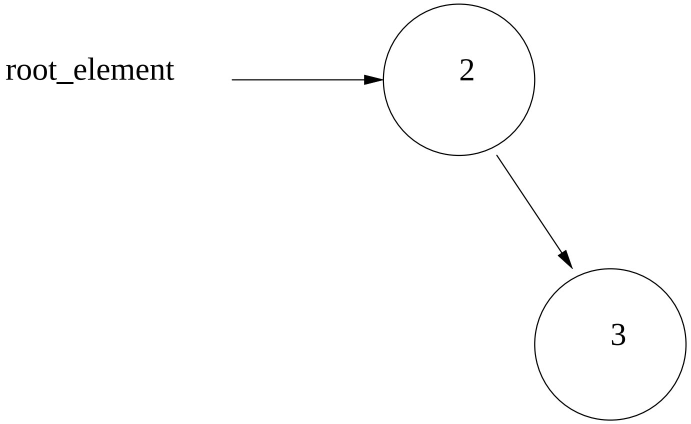
Empty tree

root_element 

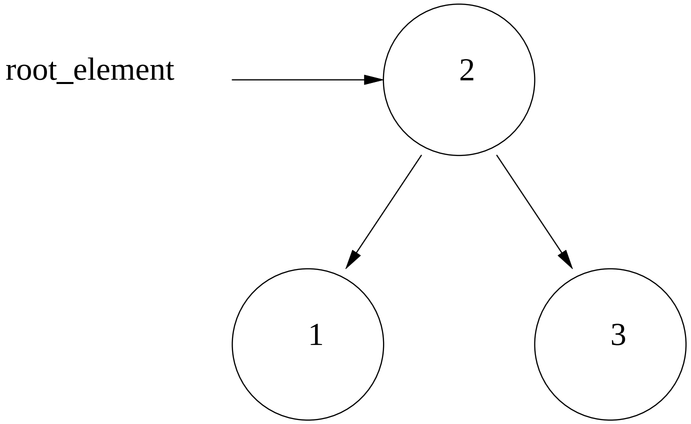
Adding 2 to the tree



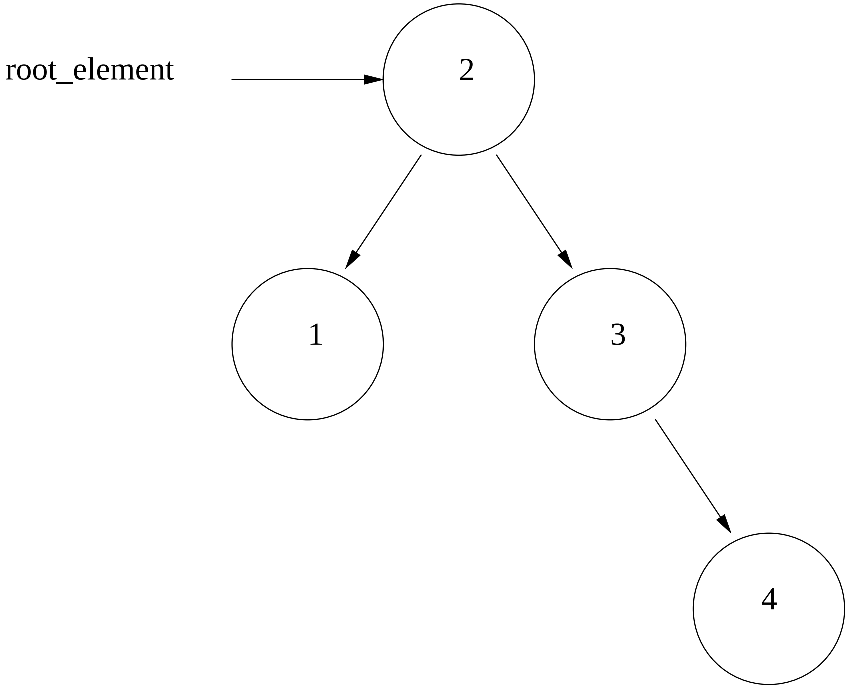
Adding 3 to the tree



Adding 1 to the tree



Adding 4 to the tree



tree organisation

- we note that if we were to add numbers in a different order the tree would be populated differently

- yet the tree still maintains the property expressed earlier

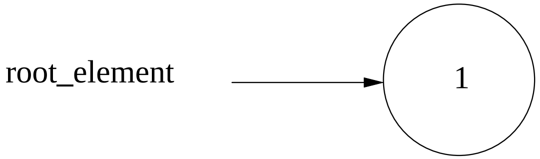
Example 2

- let us add: `1`, `2`, `3` and `4` to a binary tree

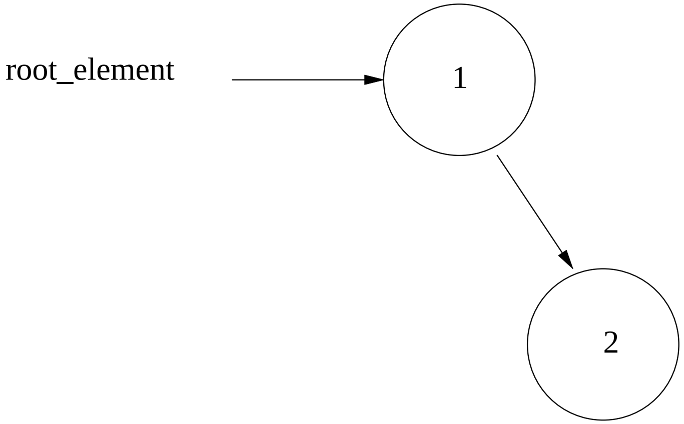
Empty tree

root_element 

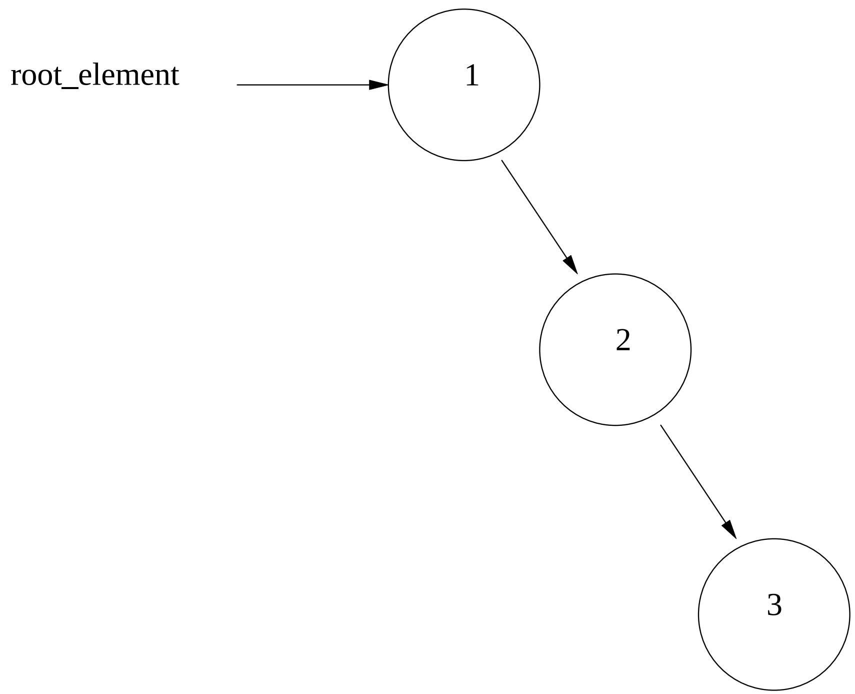
Adding 1 to the tree



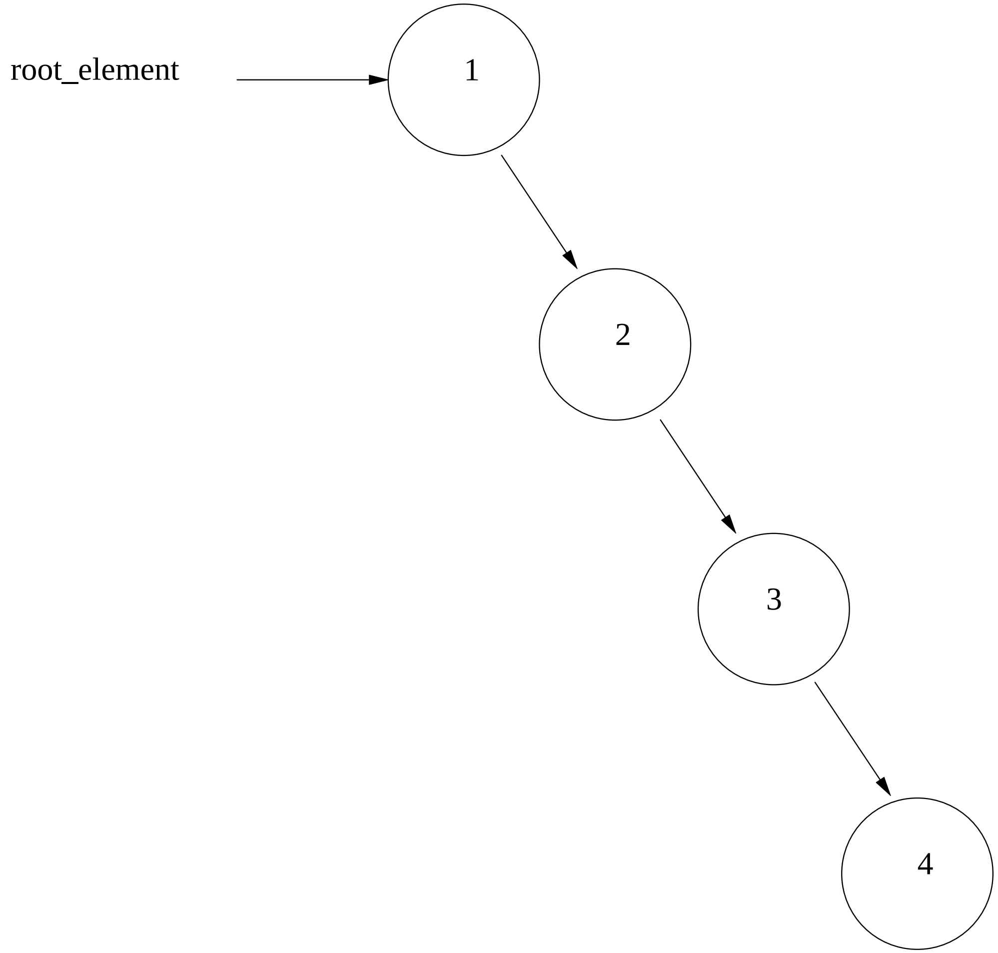
Adding 2 to the tree



Adding 3 to the tree



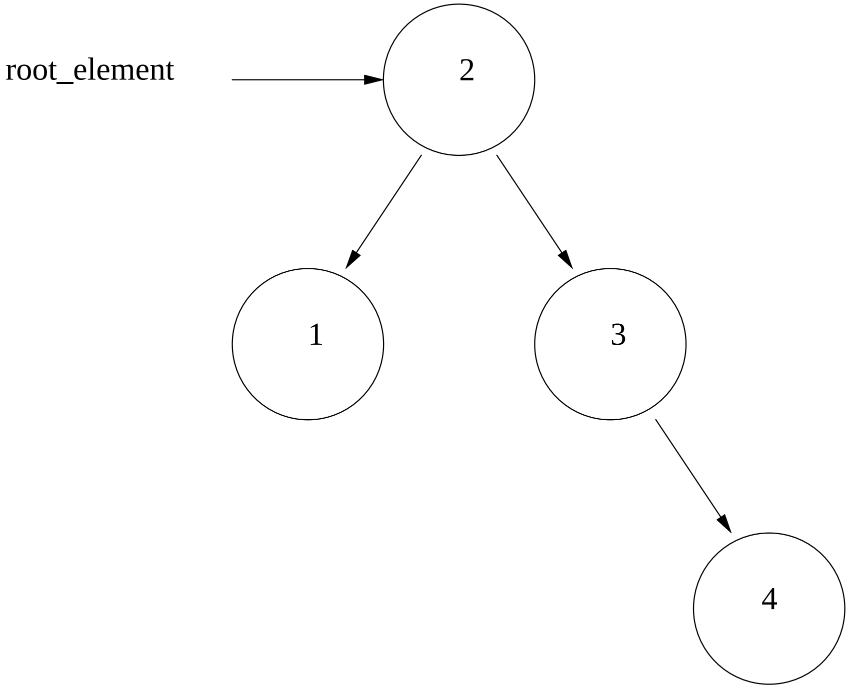
Adding 4 to the tree



Printing trees

- there are three common methods of tree traversal:
  - (when do we print the root value?)
- preorder
  - **root**, left, right
- inorder
  - left, **root**, right
- postorder
  - left, right, **root**

Example of order



- preorder: 2, 1, 3, 4
- inorder: 1, 2, 3, 4
- postorder: 1, 4, 3, 2

class element definition

examples/c++/trees/int/tree.h

```
class element
{
 public:
  element *left;
  element *right;
  int      data;
};
```

class tree definition

examples/c++/trees/int/tree.h

```
class tree
{
 private:
  element *root_element;
  friend std::ostream& operator<< (std::ostream& os, const tree& l);
  // helper methods
  int  no_of_items (element *e);
  int  height (element *e);
  void delete_element (element *e);
  element *duplicate_elements (element *e);
  void insert (element **e, int i);
  tree cons (int i, element *l, element *r);
```

class tree definition

examples/c++/trees/int/tree.h

```
  tree (void);
  ~tree (void);
  tree (const tree &from);
  tree& operator= (const tree &from);

  void inorder (std::ostream& os, element *e);
  void preorder (std::ostream& os, element *e);
  void postorder (std::ostream& os, element *e);

  tree empty (void);
  bool is_empty (void);
  tree cons (int i, tree l, tree r);
  int  root (void);
  tree cons (tree l);
  int  height (void);
  int  no_of_items (void);
  tree insert (int i);
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  tree - constructor, builds an empty list.
 *          pre-condition:    none.
 *          post-condition:   tree is created and is empty.
 */

tree::tree (void)
{
  root_element = 0;
}
```

class tree implementation

■ **examples/c++/trees/int/tree.cc**

```
/*
 *   ˜tree - deconstructor, releases the memory attached to the list.
 *           pre-condition:   none.
 *           post-condition:  tree is empty.
 */

tree::˜tree (void)
{
  delete_element (root_element);
  root_element = 0;
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  delete_element - pre-condition :  none.
 *                   post-condition:  left and right branches are deleted.
 */

void tree::delete_element (element *e)
{
  if (e != 0)
    {
      delete_element (e->left);
      delete_element (e->right);
      delete e;
    }
}
```

class tree implementation

- **examples/c++/trees/int/tree.cc**

```
/*
 *  copy operator - redefine the copy operator.
 *                  pre-condition :  a tree.
 *                  post-condition:  a copy of the tree and its elements.
 */

tree::tree (const tree &from)
{
  root_element = duplicate_elements (from.root_element);
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  operator=  - redefine the assignment operator.
 *               pre-condition :  a list.
 *               post-condition:  a copy of the list and its elements.
 *                                We delete 'this' lists elements.
 */

tree& tree::operator= (const tree &from)
{
  if (this->root_element == from.root_element)
    return *this;

  delete_element (root_element);
  root_element = duplicate_elements (from.root_element);
}
```

class tree implementation

- **examples/c++/trees/int/tree.cc**

```
/*
 *   is_empty - returns true if tree is empty.
 */

bool tree::is_empty (void)
{
  return root_element == 0;
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  empty - returns a new empty tree.
 *          pre-condition:   none.
 *          post-condition:  a new empty tree is returned.
 */

tree tree::empty (void)
{
  tree *t = new tree;
  return *t;
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  cons - concatenate i to tree.
 *         pre-condition:   none.
 *         post-condition:  returns a tree which has the
 *                          contents of, l, and, r, as its branches.
 */

tree tree::cons (int i, tree l, tree r)
{
  return cons (i,
               duplicate_elements (l.root_element),
               duplicate_elements (r.root_element));
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  cons - concatenate i to tree.
 *         pre-condition:   none.
 *         post-condition:  returns a tree which has the
 *                          contents of, l, and, r, as its branches.
 */

tree tree::cons (int i, element *l, element *r)
{
  tree *t = new tree;
  t->root_element = new element;

  t->root_element->data = i;
  t->root_element->left = l;
  t->root_element->right = r;
  return *t;
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *   root - returns the data at the root of the tree.
 *          pre-condition :  tree is not empty.
 *          post-condition:  data at the front of the list is returned.
 *                           tree is unchanged.
 */

int tree::root (void)
{
  assert (! is_empty());
  return root_element->data;
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  no_of_items - return the number of items in the tree.
 *                pre-condition : none.
 *                post-condition: returns an integer indicating
 *                                the number of items in the tree.
 *
 */

int tree::no_of_items (void)
{
  return no_of_items (root_element);
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  height - pre-condition : none:
 *           post-condition: returns the max height between
 *                           the left and right branches.
 */

int tree::height (element *e)
{
  if (e == 0)
    return 0;
  else
    return 1 + max (height (e->left), height (e->right));
}
```

class tree implementation

examples/c++/trees/int/tree.cc

```
/*
 *  insert - places, i, into the tree in the correct place.
 *           pre-condition :  none.
 *           post-condition:  if i < data then i is stored to the left
 *                            else i is stored to the right.
 */

tree tree::insert (int i)
{
  if (is_empty ())
    return cons (i, 0, 0);
  else
    insert (&root_element, i);
  return *this;
}
```

class tree implementation

■ **examples/c++/trees/int/tree.cc**

```
void tree::insert (element **e, int i)
{
  if ((*e) == 0)
    {
      (*e) = new element;
      (*e)->left = 0;
      (*e)->right = 0;
      (*e)->data = i;
    }
  else
    {
      if (i < (*e)->data)
        insert (&(*e)->left, i);
      else
        insert (&(*e)->right, i);
    }
}
```