

Collision prediction between circles

- de-tour
 - recall we can predict the time a point travels a distance by:

- $s = ut + \frac{1}{2} at^2$

- | |
|----------------------|
| s = distance |
| u = initial velocity |
| a = acceleration |
| t = time |

De-tour

- if we were to drop a small ball bearing from 1 metre, how long would it take to hit the ground?
- initial velocity ($u = 0$)
- acceleration approx ($a = 10$) ms^2

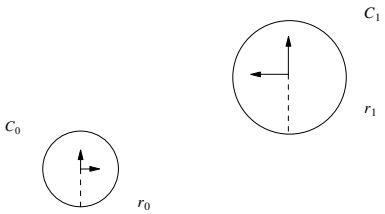
De-tour

- thus
- $1 = \frac{1}{2} at^2$
- $1 = \frac{1}{2} 10t^2$
- $1 = 5t^2$
- $\frac{1}{5} = t^2$
- $t = \pm \sqrt{\frac{1}{5}}$

De-tour

- $t = 0.45$
- $t = -0.45$
- we also note that for more complicated quadratic equations we can use the formula:
- $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- for any equation matching: $at^2 + bt + c = 0$

Collision prediction between circles

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- each circle has a radius, position, velocity and acceleration
 - radius is a scalar, all others are vectors

Collision prediction between circles

- remember that when these circles collide the distance between the two circle mid points will be $r_0 + r_1$
- we know generally that the distance between the circles can be calculated as:

$$\sqrt{(c_{0x} - c_{1x})^2 + (c_{0y} - c_{1y})^2}$$
- so we need to find the time when:

$$r_0 + r_1 = \sqrt{(c_{0x} - c_{1x})^2 + (c_{0y} - c_{1y})^2}$$

Collision prediction between circles

- using the formula for initial position, velocity and acceleration: $s = s_0 + ut + \frac{at^2}{2}$ it is also known that the position circle c_0 at time, t , is:

$$\left[c_{0x} + v_{0x}t + \frac{a_{0x}t^2}{2}, c_{0y} + v_{0y}t + \frac{a_{0y}t^2}{2} \right]$$

- correspondingly the position circle c_1 at time, t , is:

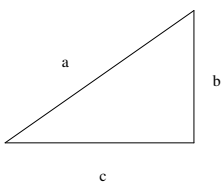
$$\left[c_{1x} + v_{1x}t + \frac{a_{1x}t^2}{2}, c_{1y} + v_{1y}t + \frac{a_{1y}t^2}{2} \right]$$

Collision prediction between circles

- recall Pythagorean theorem can be used to find the length of the hypotenuse from the length of the base and height of a right angled triangle

Collision prediction between circles

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- $a^2 = b^2 + c^2$
- $a = \sqrt{b^2 + c^2}$

Collision prediction between circles

- a is the radius of circle 0 and circle 1
 - $a = r_0 + r_1$
- b is the difference of the y-axis position of the circles
- c is the difference of the x-axis position of the circles

Collision prediction between circles

- therefore we need to find the time at which the distance between both circles is $r_0 + r_1$ which is:

$$r_0 + r_1 = \sqrt{\left[\left(c_{0x} + v_{0x}t + \frac{a_{0x}t^2}{2} \right) - \left(c_{1x} + v_{1x}t + \frac{a_{1x}t^2}{2} \right) \right]^2 + \left[\left(c_{0y} + v_{0y}t + \frac{a_{0y}t^2}{2} \right) - \left(c_{1y} + v_{1y}t + \frac{a_{1y}t^2}{2} \right) \right]^2}$$

- which can be easily rearranged as:

$$0 = \left[\left(c_{0x} + v_{0x}t + \frac{a_{0x}t^2}{2} \right) - \left(c_{1x} + v_{1x}t + \frac{a_{1x}t^2}{2} \right) \right]^2 + \left[\left(c_{0y} + v_{0y}t + \frac{a_{0y}t^2}{2} \right) - \left(c_{1y} + v_{1y}t + \frac{a_{1y}t^2}{2} \right) \right]^2 - (r_0 + r_1)^2$$

Collision prediction between circles

- we need to multiply the equation and separate out the values: A, B, C, D, E where:

$$At^4 + Bt^3 + Ct^2 + Dt + E = 0$$

- then we solve for, t and the smallest value of t which is greater than zero is the next collision time

- see `pge/twoDsim.mod` in the function `findCollisionCircles`

wxmaxima

- fortunately, we have a tool called `wxmaxima` (similar to Mathematica) which will manipulate the algebra without mistake :-)

wxmaxima

```

ri + rj == sqrt(abs(xin-xjn)^2 + abs(yin-yjn)^2)    for values of t
ri + rj == sqrt(((xi + vxi * t + aix * t^2 / 2.0) - (xj + vxj * t + ajx * t^2 / 2.0) - (yi + vyi * t + aiy * t^2 / 2.0) - (yj + vyj * t + ajy * t^2 / 2.0))^2)

let:
a = xi
b = xj
c = vxi
d = vxj
e = aix
f = ajx
g = yi
h = yj
k = vyi
l = vyj
m = aiy
n = ajy
o = ri
p = rj
t = t
    
```

wxmaxima

```
o + p == sqrt(((a + c * t + e * t^2 / 2.0) - (b + d * t + f * t^2 / 2.0) - (g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0)) / ((g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0))
o + p == sqrt(((a + c * t + e * t^2 / 2.0) - (b + d * t + f * t^2 / 2.0) - (g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0)) / ((g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0))
0 == ((a + c * t + e * t^2 / 2.0) - (b + d * t + f * t^2 / 2.0) - (g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0)) / ((g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0))
now using wxmaxima
expand; factor; ratsimp
```

wxmaxima

```
p+o == (sqrt((n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*l)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
((4*h-4*g)*n+(4*g-4*h)*m+4*l^2-8*k*l+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
((8*h-8*g)*l+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2))/2
2*(p+o) == (sqrt((n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*l)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
((4*h-4*g)*n+(4*g-4*h)*m+4*l^2-8*k*l+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
((8*h-8*g)*l+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2))
```

wxmaxima

```
(2*(p+o))^2 == ((n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*l)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
((4*h-4*g)*n+(4*g-4*h)*m+4*l^2-8*k*l+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
((8*h-8*g)*l+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2)
0 == (n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*l)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
((4*h-4*g)*n+(4*g-4*h)*m+4*l^2-8*k*l+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
((8*h-8*g)*l+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2)-
((2*(p+o))^2)
```

wxmaxima

```
solve polynomial:
A := sqrt(n)-2.0*m*sqrt(m)+sqrt(e)-2.0*e*sqrt(e) ;
B := (4.0*1-4.0*k)*n+(4.0*k-4.0*l)*m+(4.0*d-4.0*c)*f+(4.0*c-4.0*d)*e ;
C := (4.0*h-4.0*g)*n+(4.0*g-4.0*h)*m+4.0*sqrt(l)-8.0*k*l+4.0*sqrt(k)+
(4.0*b-4.0*a)*f+(4.0*a-4.0*b)*e+4.0*sqrt(d)-8.0*c*d+4.0*sqrt(c) ;
D := (8.0*h-8.0*g)*l+(8.0*g-8.0*h)*k+(8.0*b-8.0*a)*d+(8.0*a-8.0*b)*c ;
E := 4.0*sqrt(h)-8.0*g*sqrt(g)+4.0*sqrt(g)+4.0*sqrt(b)-8.0*a*sqrt(a)-sqrt(2.0*(p+o))
```

fortunately, we can cut and paste the output from wxmaxima into our code and carefully convert it into program code and it works!

Conclusion

- we have seen that we can predict when two circles will collide in the future
 - using x and y components based on $s = ut + a \frac{t^2}{2}$

- in the next lecture we will extend this to look at calculating the time a circle hits a polygon