

**Collision prediction between circles**

- de-tour
  - recall we can predict the time a point travels a distance by:

- $s = ut + \frac{1}{2} at^2$

- |  |
|--|
| s = distance<br>u = initial velocity<br>a = acceleration<br>t = time |
|--|

slide 3  
gaius**De-tour**

- if we were to drop a small ball bearing from 1 metre, how long would it take to hit the ground?
- initial velocity ( $u = 0$ )
- acceleration approx ( $a = 10$ )  $ms^2$

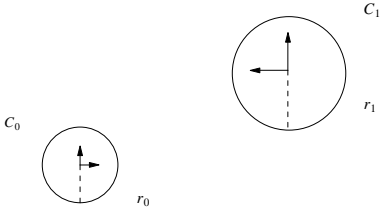
slide 4  
gaius**De-tour**

- thus
- $1 = \frac{1}{2} at^2$
- $1 = \frac{1}{2} 10t^2$
- $1 = 5t^2$
- $\frac{1}{5} = t^2$
- $t = \pm \sqrt{\frac{1}{5}}$

**De-tour**

- $t = 0.45$
- $t = -0.45$
- we also note that for more complicated quadratic equations we can use the formula:
- $$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- for any equation matching:  $at^2 + bt + c = 0$

**Collision prediction between circles**

- 
- each circle has a radius, position, velocity and acceleration
  - radius is a scalar, all others are vectors

**Collision prediction between circles**

- remember that when these circles collide the distance between the two circle mid points will be  $r_0 + r_1$
- we know generally that the distance between the circles can be calculated as:
 
$$\sqrt{(c_{0x} - c_{1x})^2 + (c_{0y} - c_{1y})^2}$$
- so we need to find the time when:
 
$$r_0 + r_1 = \sqrt{(c_{0x} - c_{1x})^2 + (c_{0y} - c_{1y})^2}$$

**Collision prediction between circles**

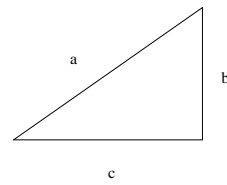
- using the formula for initial position, velocity and acceleration:  $s = s_0 + ut + \frac{at^2}{2}$  it is also known that the position circle  $c_0$  at time,  $t$ , is:
 
$$\left[ c_{0x} + v_{0x}t + \frac{a_{0x}t^2}{2}, c_{0y} + v_{0y}t + \frac{a_{0y}t^2}{2} \right]$$
- correspondingly the position circle  $c_1$  at time,  $t$ , is:
 
$$\left[ c_{1x} + v_{1x}t + \frac{a_{1x}t^2}{2}, c_{1y} + v_{1y}t + \frac{a_{1y}t^2}{2} \right]$$

## Collision prediction between circles

- recall Pythagorean theorem can be used to find the length of the hypotenuse from the length of the base and height of a right angled triangle

## Collision prediction between circles

■



- $a^2 = b^2 + c^2$

- $a = \sqrt{b^2 + c^2}$

## Collision prediction between circles

- $a$  is the radius of circle 0 and circle 1
  - $a = r_0 + r_1$
- $b$  is the difference of the y-axis position of the circles
- $c$  is the difference of the x-axis position of the circles

## Collision prediction between circles

- therefore we need to find the time at which the distance between both circles is  $r_0 + r_1$  which is:

- $$r_0 + r_1 = \sqrt{\left[ \left( c_{0x} + v_{0x}t + \frac{a_{0x}t^2}{2} \right) - \left( c_{1x} + v_{1x}t + \frac{a_{1x}t^2}{2} \right) \right]^2 + \left[ \left( c_{0y} + v_{0y}t + \frac{a_{0y}t^2}{2} \right) - \left( c_{1y} + v_{1y}t + \frac{a_{1y}t^2}{2} \right) \right]^2}$$

- which can be easily rearranged as:

■

- $$0 = \left[ \left( c_{0x} + v_{0x}t + \frac{a_{0x}t^2}{2} \right) - \left( c_{1x} + v_{1x}t + \frac{a_{1x}t^2}{2} \right) \right]^2 + \left[ \left( c_{0y} + v_{0y}t + \frac{a_{0y}t^2}{2} \right) - \left( c_{1y} + v_{1y}t + \frac{a_{1y}t^2}{2} \right) \right]^2 - (r_0 + r_1)^2$$

## Collision prediction between circles

## wxmaxima

- we need to multiply the equation and separate out the values: A, B, C, D, E where:
- $At^4 + Bt^3 + Ct^2 + Dt + E = 0$
- then we solve for,  $t$  and the smallest value of  $t$  which is greater than zero is the next collision time
- see `pge/twoDsim.mod` in the function `findCollisionCircles`

- fortunately, we have a tool called `wxmaxima` (similar to Mathematica) which will manipulate the algebra without mistake :-)

## wxmaxima

## wxmaxima

```

r_i + r_j == sqrt(abs(x_i - x_j)^2 + abs(y_i - y_j)^2) for values of t
r_i + r_j == sqrt(((x_i + v_xi * t + a_ix * t^2 / 2.0) - (x_j + v_xj * t + a_jx * t^2 / 2.0))^2 + ((y_i + v_yi * t + a_iy * t^2 / 2.0) - (y_j + v_yj * t + a_jy * t^2 / 2.0))^2)

let:
a = x_i
b = x_j
c = v_xi
d = v_xj
e = a_ix
f = a_jx
g = y_i
h = y_j
k = v_yi
l = v_yj
m = a_iy
n = a_jy
o = r_i
p = r_j
t = t
    
```

```

o + p == sqrt(((a + c * t + e * t^2 / 2.0) - (b + d * t + f * t^2 / 2.0))^2 + ((g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0))^2)

o + p == sqrt(((a + c * t + e * t^2 / 2.0) - (b + d * t + f * t^2 / 2.0))^2 + ((g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0))^2)

0 == ((a + c * t + e * t^2 / 2.0) - (b + d * t + f * t^2 / 2.0))^2 + ((g + k * t + m * t^2 / 2.0) - (h + l * t + n * t^2 / 2.0))^2 - (o + p)^2

now using wxmaxima
expand ; factor ; ratsimp
    
```

wxmaxima

```
p+o == (sqrt((n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*1)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
(4*h-4*g)*n+(4*g-4*h)*m+4*1^2-8*k*1+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
(8*h-8*g)*1+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2))/2

2*(p+o) == (sqrt((n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*1)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
(4*h-4*g)*n+(4*g-4*h)*m+4*1^2-8*k*1+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
(8*h-8*g)*1+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2))
```

wxmaxima

```
(2*(p+o))^2 == (n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*1)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
(4*h-4*g)*n+(4*g-4*h)*m+4*1^2-8*k*1+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
(8*h-8*g)*1+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2)

0 == (n^2-2*m*n+m^2+f^2-2*e*f+e^2)*t^4+
((4*1-4*k)*n+(4*k-4*1)*m+(4*d-4*c)*f+(4*c-4*d)*e)*t^3+
(4*h-4*g)*n+(4*g-4*h)*m+4*1^2-8*k*1+4*k^2+(4*b-4*a)*f+
(4*a-4*b)*e+4*d^2-8*c*d+4*c^2)*t^2+
(8*h-8*g)*1+(8*g-8*h)*k+(8*b-8*a)*d+(8*a-8*b)*c)*t+
4*h^2-8*g*h+4*g^2+4*b^2-8*a*b+4*a^2)-
(2*(p+o))^2
```

wxmaxima

```
solve polynomial:
A := sqrt(n)-2.0*m*n+sqrt(m)+sqrt(f)-2.0*e*f+sqrt(e) ;
B := (4.0*1-4.0*k)*n+(4.0*k-4.0*1)*m+(4.0*d-4.0*c)*f+(4.0*c-4.0*d)*e ;
C := (4.0*h-4.0*g)*n+(4.0*g-4.0*h)*m+4.0*sqrt(1)-8.0*k*1+4.0*sqrt(k)+
(4.0*b-4.0*a)*f+(4.0*a-4.0*b)*e+4.0*sqrt(d)-8.0*c*d+4.0*sqrt(c) ;
D := (8.0*h-8.0*g)*1+(8.0*g-8.0*h)*k+(8.0*b-8.0*a)*d+(8.0*a-8.0*b)*c ;
E := 4.0*sqrt(h)-8.0*g*h+4.0*sqrt(g)+4.0*sqrt(b)-8.0*a*b+4.0*sqrt(a)-sqrt(2.0*(p+o))
```

Conclusion

- fortunately, we can cut and paste the output from wxmaxima into our code and carefully convert it into program code and it works!

- we have seen that we can predict when two circles will collide in the future
  - using  $x$  and  $y$  components based on  $s = ut + a \frac{t^2}{2}$
- in the next lecture we will extend this to look at calculating the time a circle hits a polygon